Computing Examples for the

CURTA

Calculating Machine

CONTINA AG, VADUZ

PRINCIPALITY OF LIECHTENSTEIN (ECONOMIC AND CUSTOMS UNION WITH SWITZERLAND)

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Introduction

These computing examples complete the short instructions sheet "YOUR CURTA CALCULATOR" which accompanies every CURTA machine, and **it is assumed that the use of the machine for effecting the four arithmetical rules, as described therein, is understood.**

Every example in this collection relates to both CURTA models, with the exception of those which are marked "only CURTA Model II". The two machines differ only in the number of figures which they can handle. These are:

	Setting Register	Counter Register	Result Register
for the CURTA Model I	8	6	11
for the CURTA Model II	11	8	15

In every example the following abbreviations are used:

S.R. = Setting Register;
C.R. = Counter Register;
R.R. = Result Register.

The expression "Machine ready" signifies that

1. S.R., C.R. and R.R. have been cleared;
2. The operating handle is in its zero stop position;
3. The carriage is in position 1;
4. The reversing lever is in its normal (upper) position.

The following list of contents will give the reader a survey of the subjects considered.

CONTINA AG.,
Vaduz / Liechtenstein

CONTENTS

Division by breaking down

(subtractive method)

$$\frac{a + b + c}{d} ; \frac{9526.8 + 37.51 + 645.62}{3} = ?$$

(5 or 6 figures are required in the answer.) Machine ready.

To begin with it is necessary to add together the three terms in the numerator. In order to accumulate the sum as far to the left as possible in the R.R., it is necessary to move the carriage as far to the right as possible. But care must be taken to move the carriage only so far to the right that space is left to accommodate the most significant figures. Put the carriage in position 5. Set decimal points between positions 2 and 3 in S.R., and positions 6 and 7 in R.R.

Step 1. Set the first term of the sum on the right hand side of S.R. (00 ... 9526.80). One plus turn of the operating handle.

Step 2. Set 00 ... 37.51 in S.R. One plus turn.
Step 3. Third term in S.R. One plus turn. Sum in R.R. is now 10209.93. **Clear only the C.R. and S.R.**

Now follows the division by 3 by the breaking down method:

Carriage in position 6. Set the divisor 3 in S.R. as far to the left as possible, but not so far that a minus turn would cause a negative number to be indicated in R.R.; i. e. set 3 in position 5. Place a decimal point between positions 4 and 5 of S.R.

Place the **reversing lever in its lower position.**

With the operating handle pulled upwards we continue to subtract the contents of S.R. from the contents of R.R. until the latter indicates a negative number, i. e. until a **row of 9's** appears (or possibly a row of 9's and an 8); thus turn the handle in its minus position and **observe R.R.** After 4 minus turns R.R. indicates 9 ... 8209930. Now make one **plus turn;** a row of 0's now appears in R.R., and the number in R.R. is in fact 0 ... 1 209.930 ... 0. This number is the remainder after dividing the original dividend by the number now in C.R., i. e. 3, which is the most significant figure in the quotient.

Set the carriage in the next lower position, i. e. position 5. Again turn the handle in its minus position until a row of 9's appears in R.R., which occurs in this case after 5 turns. Make **one plus turn.**

Carriage in position 4. Turn the handle in its minus position until a row of 9's appears in R.R. as described previously, and again one plus turn.

Carriage in position 3. 4 minus turns, 1 plus turn.

Carriage in position 2. 4 minus turns, 1 plus turn.

Carriage in position 1. Make one minus turn and zero appears in R.R., which indicates that the division is at an end. (In the case of a division in which there is a remainder, the remainder appears in R.R.) Check that the number 3 (the divisor) is in S.R.

From the rule concerning decimal points in a division*) we have 6 (R.R.) — 4 (S.R.) = 2 decimal places in C.R. Thus we set a decimal point between positions 2 and 3 of C.R. and read the **result** (i. e. the quotient) **as 3403.31.** Return the reversing lever to its normal position. By using a CURTA

*) See short instructions sheet "Your CURTA calculator".

Model II (which has at its disposal 8 places in C.R.) a quotient of 8 significant figures may be obtained.

Division by multiplying by a reciprocal

(If we require a number of dividends to be divided by the same divisor we can set the divisor as a constant number in S.R. and build up the successive dividends in R.R., without resetting the divisor in S.R. and clearing C.R. and R.R. each time. The successive quotients appear in C.R. and can be noted down. See the costing example on p. 28.)

If, however, **a large number** of dividends are to be divided by the same divisor, it is simplest to calculate the reciprocal of the divisor once and for all, and place this quantity as a **constant multiplier** in S.R. The divisions are then transformed into multiplications, e. g.

$$1633 \div 11.7$$
$$341.5 \div 11.7$$
$$67.8 \div 11.7$$

The division 1 ÷ 11.7 to 6 or 7 figures gives us the reciprocal of the divisor 11.7, namely 0.0854701. We place this number in the right hand end of S.R., clear C.R. and R.R., and set a decimal point between 7 and 8 of S.R. The three divisions may now be carried out as 3 normal multiplications, namely,

$$0.0854701 \times 1633 = 139.572$$
$$0.0854701 \times 341.5 = 29.1880$$
$$0.0854701 \times 67.8 = 5.79487$$

An experienced computor will dispense with the intermediate clearing of C.R. and R.R. after each multiplication, and will build up the contents of C.R. to each successive multiplier by appropriate plus and minus turns of the handle.

The rule of three

$$\frac{a \times b}{c} \; ; \; \frac{180 \times 46}{144} = ?$$

First Method (subtractive division)
Machine ready.

First we form the product 180 × 46 and then divide by 144. The multiplication 180 × 46 is performed in such a way that the product appears as far to the left as possible in R.R.

Set the number 46 in S.R. by means of knobs 2 and 1. With the carriage in position 6 make 2 plus turns and in position 5 2 minus turns (short-cut multiplication). **The product 8280.000** is now in R.R. (with 3 decimal places in accordance with the decimal rule).

Now follows the division 8280 ÷ 144 by the subtractive method (see the previous example). Clear **only C.R.** Set the number 144 in the right hand of S.R. Place the **reversing lever in the lower position.** The process of subtractive division now takes place with the carriage starting in position 5. In this case the division produces no remainder, and the **result** (quotient) **in C.R. is 57.5.**
Return the reversing lever to its normal position.

Second Method

In numerous calculations involving the rule of three the calculation of the intermediate quantity $\frac{a}{c}$ is also required; e. g.

If certain articles cost 180 francs/gross, how much does each article cost, and how much do 46 articles cost?
Machine ready.

First we must calculate the cost of one article (180 ÷ 144) by additive division, so the number 144 is set in S.R. by means of knobs 3—1, and with the carriage in position 6, the dividend 180 is built up figure by figure in R.R. by positive turns of the operating handle. The quotient appearing in C.R. is **1.25 = cost of one article.**

Since we are about to multiply the cost of one article (1.25) by 46, and the number 1.25 is already in C.R., clear **only R.R.** Set the number 46 in the right hand end of S.R. and place the **reversing lever in the lower position.**

Starting with the carriage in position 4, reduce every figure in C.R. to zero with plus turns of the handle. This indicates a multiplication by 1.25, and the reduction of the contents of C.R. to zero is a check. (See the examples on pages 12, 34, 43.) **The result** in R.R. is 57.50 francs, namely the price of 46 articles. (Return the reversing lever to its normal position.)

The rule of three in a single calculation

(only CURTA Model II)

In those cases in which the numbers involved are given only to a small number of figures, the rule of three may be effected at one blow by using a CURTA Model II, the quotient being obtained with 4 or 5 figures.

$$\frac{1764 \times 375}{144}$$

Machine ready.

The division 1764 ÷ 144 is carried out at the left hand end of R.R. by the additive method. At the same time the quotient of this division is multiplied by 375 in the right hand end of R.R. S.R. 1440000375.

Place the carriage in position 5 and by turning the operating handle in the appropriate manner build up the number 1764 figure by figure in R.R. R.R. now reads 176400004593750 and C.R. 00012250. 1764 is the dividend 12.25 is the quotient 1764 ÷ 144 and **4593.75 is the final result** (i. e. the product of 12.25 and 375).

$$\frac{19.45 \times 87.2}{34.4}$$

Machine ready.

S.R. 34.40000087.2. Carriage in position 4. (Since the first figure of the dividend is smaller than the first figure of the divisor room for one more digit [i. e. position 15] in R.R., must be left free.)

Build up the number 19.45 from the number 34.4.

R.R. 19.44976 0 **49.30(288).** C.R. 000.5654.

Extended rule of three $\frac{a \times b \times c}{d \times e \times f}$

$$\frac{325 \times 677}{12 \times 119} = ?$$

Firstly the division 325 ÷ 12 is carried out by the additive method to give 5 figures in the quotient; the division is commenced with the carriage in position 5. The quotient 27.083 appears in the first 5 figures of C.R. **Clear only R.R.** Set 677 in the right hand end of S.R. Place the **reversing lever to its lower position.** Starting with the carriage in position 1 reduce the number in C.R. to zero by plus turns of the handle (see p. 11). The **product 18335.191** appears in R.R.

Carriage in position 6. Set the new divisor 119 at the right hand end of S.R. Carry out the division by means of the subtractive method as far as position 1 of the carriage.

R.R. now indicates a remainder of 0.028, C.R. indicates the **quotient 154.077.** (Replace the reversing lever in its normal position.) By using this method it is possible that a small rounding off error can occur which, however, is insignificant in most practical cases. If it is required to eliminate this error completely, it is recommended that first the factors in the denominator be multiplied together and noted down, and secondly the factors in the numerator be multiplied together with the product being acuumulated as far to the left as is possible in R.R. The divisor may then be set in S.R. and the division carried out by the subtractive method.

Calculation of roots

Square Roots

(Hermann's Method)

In the method to be described it is supposed, that by means of a slide rule or auxiliary tables or by judicious guessing, an approximate square root has been found. We wish to obtain a better approximation. Let N be the approximate value of R, the square root of R^2, and denote the error in the approximation by E, so that $R = N + E$. The method proceeds by setting N in S.R., multiplying by N (which appears in C.R.) to produce N^2 in R.R. The quantity 2 N is then set in S.R. (without clearing R.R. or C.R.), and R^2 is built up from N^2 in R.R. Since

$$R^2 = N^2 + 2\,NE + E^2$$

it follows that (if we neglect E^2) E is added to C.R. Since C.R. already contained N, C.R. now reads $N + E$, the new approximation.

$$\sqrt{150} = ?$$

Initial approximation = 12.2.

Machine ready.

Carriage position 6. Set 12.2 in S.R. with knobs 3 to 1. Multiply by 12.2 starting from position 6.

Carriage position 6 — 1 plus turn.
Carriage position 5 — 2 plus turns.
Carriage position 4 — 2 plus turns.

Set a decimal point between position 6 and 5 of R.R.: **148.84000.**

S.R. Set twice the initial approximation (i. e. 24.4) in the same positions as were used for the initial approximation before. By turning the handle in the appropriate manner in consecutive positions of the carriage, increase the number in R.R. to 150, thus

Carriage position 4 — 1 plus turn.
Carriage position 3 — 5 minus turns.
Carriage position 2 — 3 minus turns.
Carriage position 1 — 5 plus turns.

R.R. now reads 149.999.

C.R. now gives **to 6 correct figures the root 12.2475.**

This method determines as many additional correct figures as there were correct figures in the approximation. In this case the initial approximation is correct to 3 figures, and the desired approximation to 6 figures. From this rule it is unnecessary to proceed further; furthermore we have in this particullar case come very near to the required number (150) with 149.999.

If one requires 8 correct figures in the root and a CURTA Model II is available, one can repeat the previous example using the initial approximation 12.25 (i. e. with 4 correct figures).

One then obtains the root

12.247449

in C.R.

Remark: The rule that one gains as many correct figures in the root as one has to start with is capable of a limited number of exceptions.

(See p. 49 same method by subtraction.)

Cube roots

CURTA users who have at their disposal our CURTA Tables may, by use of these tables, obtain a cube root correct to 5 figures by means of an addition and one subsequent multiplication. These tables will be forwarded free of charge on application to our headquarters.

Use of tables can be avoided by using an extension of the method for obtaining square roots described in the last section.

Let N be the approximate value of R, the cube root of R^3, and denote the error in the approximation by E, so that $R = N + E$, and

$$R^3 = N^3 + 3\,N^2\,E + \text{terms in } E^2 \text{ and } E^3.$$

The computation proceeds by arriving at a situation in which we have N^3 in R.R., $3\,N^2$ in S.R., and N in C.R. We then build up the contents of R.R. to R^3, i. e. from the above equation, adding approximately E to the contents of C.R. C.R. thus contains a quantity which approximates to $N + E$.

It will be appreciated that (and this remark applies equally well to the method for deriving square roots) if the derived approximation is insufficiently accurate, it may be improved by repeating the process.

$$\sqrt[3]{132.651} = ?$$

Machine ready.

Initial approximation = 5.0.

Carriage in position 6: Set 5 in S.R. with knob 1.

Multiply by 5 in position 6.

R.R.: 25. Set a decimal point in C.R. between positions 5 and 6.

Clear C.R. and R.R. Set 25 in S.R. with knobs 2 and 1.

Multiply by 5 in position 6. Replace 25 in S.R. by 75 ($= 3 \times 25$).

Increase the contents of R.R. to 132.651, thus

> Carriage in position 5: 1 plus turn.
> Carriage in position 3: 2 plus turns.
> Carriage in position 1: 1 plus turn.

R.R. now contains 132.65075. C.R. contains 5.10201. The cube root of 132.651 is 5.1. If the process is repeated with the first approximation 5.10201, C.R. finally contains 5.1.

Further roots may sometimes be computed by application of the preceding methods,

e. g. $\sqrt[6]{14} = ?$

Firstly we compute the square root 3.74166 and then the cube root of this number, namely

$$\sqrt[3]{3.74166} = 1.5524 = \sqrt[6]{14}$$

Continued multiplication

a × b × c × d × . . . etc.

Introductory Remark: The limit of a computation of this type is reached when the number of figures in the product is equal to the number of figures in the Result Register, i. e. 11 figures for the CURTA Model I and 15 figures for the CURTA Model II. Before the computation is begun it is possible to estimate the number of figures in the product by adding together the number of figures in the different factors. One can, if the need arises, omit the last figures of the partial products in order to avoid an excessively long end product.

Obviously one can compute a function of the form a × b × c × d × . . . etc. by multiplying the first two factors together, setting the product in S.R., multiplying by the third factor, and so on. Use of this method, however, means that each partial product must be set at each stage in S.R. We now describe two methods which may readily be applied in many cases.

First Method

38 × 24 × 57 × 63.44 = ?

Machine ready.

I. S.R.: Set the number 38 with knobs 2 and 1.
C.R.: Develop 24 (normal multiplication).
R.R.: Result 912 = Partial product I.

II. S.R.: Set the next factor diminished by a tenth, i. e. 56.9 = (57—1/10) with knobs 3 to 1.

Carriage: Place the last figure on the right of S.R. immediately under the first figure on the left of the partial product in R.R., thus move the carriage to position 3.

Handle: Continue to turn the operating handle in its plus position until the number immediately above the 9 in S.R. goes to 0. One thus observes this number in R.R. and continues to turn the handle until it reads 0 (optical control). 513012 now appears in R.R. Carriage: Position 2. One observes the 2nd place in R.R.; after one plus turn this indicates a 0. R.R. now reads 518702.

Carriage: Position 1. One observes the first figure of R.R.; this indicates 0 after 2 plus turns. R.R. then reads 519840. The partial product II is 51984.0 (because 56.9 was set instead of 57 which gives one decimal place in R.R.). The partial product II was derived as follows: (56.9 × partial product I) + 1/10 partial product I = 57 × partial product I. III. S.R.: Set the number 63.439 = (63.44–1/10) with knobs 5 to 1.

Carriage: Position 6. Turn the handle in its plus position and at the same time observe the 6th position of R.R. When this reaches 0 R.R. reads 31720019840.

Carriage: Position 5. When the 5th number in R.R. has been reduced to zero, R.R. reads 32354409840. Proceed in this manner from position to position up to and including position 1 of the carriage. Finally R.R. indicates 32970649600. The final product, after bearing in mind the three decimal places of the number, is thus **3297864.96.**

Remark: It can occur that the number of figures in the partial product is greater than the number of positions through which the carriage may be moved. The next example shows nevertheless, that the same method can still be used. It is, however, advisable to arrange the computation in such a manner, where possible, that the factor with the largest number of figures is set last in S.R. (see the first example).

63.44 × 38 × 24 × 57 = ?

(the result is already known). The multiplication 63.44 × 38 = 2410.72 is normal. Next set 23.9 in S.R. with knobs 3 to 1.

Carriage: Position 6. By turning the handle in its plus position in positions 6, 5, 4, 3, 2 and 1 of the carriage reduce every digit in R.R. to 0. R.R. now reads 57857280. The partial product is 57857.280.

Continuation of the calculation with a CURTA Model I.

Carriage: Position 6.

S.R.: Set 56.9 with knobs 5 to 3 (knobs 1 and 2 to 0). By turning the handle in its plus position in positions 6 to 1 of the carriage reduce each digit in turn to 0 (first the 8th digit, then the 7th and so on). The 2nd and 1st digits of R.R. are not reduced to zero. R.R. now reads 3297860.4080. We can identify this result with that produced in the previous example. If complete accuracy is required for the last two figures the computation may be conducted as follows:

S.R.: Set 56.9 with knobs 3 to 1 (knobs 4 and 5 to 0).

Carriage: Position 2. By turning the handle in its plus position reduce the digits in positions 8 to 2 to 0.

Carriage: Position 1. This digit is already zero, the calculation is finished. R.R. reads **3297864.9600.**

Continuation of the calculation with a CURTA Model II

The carriage of a CURTA Model II may be moved through 8 positions, and in this instance the calculation in hand may be completed in the normal manner as described in I.

However, as an exercise for the case in which the number of figures in the partial product exceeds the number of positions through which the carriage may be moved, and in order fully to appreciate the principle described, it is advisable to carry out the exercise given for the CURTA Model I. We therefore request you to perform the previous method on your CURTA Model II. The end result in R.R. is the same as before.

Second Method

$$38 \times 24 \times 57 \times 63.44 = ?$$

Machine ready.

Proceed as for section I (page 16) of the previous example. This method differs from the previous one in that the next factor is reduced by a unit in the last figure instead of by a tenth. Thus set 57 — 1 = 56 in S.R. with knobs 2 and 1.

Clear only C.R.

Carriage: position 3. Handle: the arrow points to a 9 in R.R. Memorize this number and carry out 9 plus turns of the handle. Naturally a 9 appears in C.R.

Carriage: Position 2. The arrow now points to a 1 in R.R., thus one plus turn of the handle.

Carriage: Position 1. The arrow points to a 2 in R.R., thus 2 plus turns of the handle. R.R. shows partial product II, namely 51984, and C.R. indicates partial product I, namely 912. Set in S.R. there is the third factor minus a unit, namely 56. Partial product II was arrived at as follows: (56 × Partial product I) + (1 × Partial product I), (which was already in R.R.) = 57 × Partial product I. Clear only C.R., and set in the right hand end of S.R. the factor 63.44 with the last figure diminished by a unit, i. e. 63.43. Beginning with the carriage in position 5 turn the handle in its plus position so as to build up in C.R. the number now in R.R. **The final result** indicated in R.R. is **3297864.96.**

The advantage of this second method is that no extra zeros appear at the right hand end of R.R., and thus the full capacity of the machine can be utilised. It is, however necessary to check the calculations by noting that the partial products appear in C.R., whereas in the first method this was automatically assured by the appearance of a 0 in the successive positions of R.R.

Cubes without intermediate notes

$$327^3 = ?$$

We first obtain 327^2, i. e. 327×327 by normal multiplication. The square 106929 is built up in R.R. S.R. indicating 327 and C.R. also indicating 327 are **not** cleared.
C.R. will now be built up to 106929 starting with the carriage in position 6 in order that the least significant figures in R.R. will not be modified in the ensuing multiplications.
Carriage: Position 6. The arrow points to a 1 in R.R. In the corresponding 6th position of C.R. there is a 0; thus one plus turn.
Carriage: Position 5. The arrow points to a 0. In position 5 of C.R. there is similarly a 0. Thus this position can be skipped over.
Carriage: Position 4. The arrow points to a 6 in R.R., thus turn the handle in its plus position until C.R. also indicates 6 in this position.
Carriage: Position 3. The arrow points to a 9 in R.R. Thus turn the handle in its plus position until the 3rd position in C.R. also indicates 9.
Carriage: Position 1. In this position **both** R.R. and C.R. indicate a 2. Thus this position can be skipped over.
Carriage: Posítion 1. In this position R.R. indicates a 9, and C.R. a 7; thus 2 plus turns.
Now the basic number 327 is in S.R., in C.R. the square 106929, and **in R.R. the 3rd power 34965783.**

Higher Powers

Clearly the capacity of the machine used limits the order of the power to be computed. In the previous case for example the fourth power may be computed with a Model I by setting the cube and multiplying by 327. With the result 11433811041 the limit of the capacity of R.R. is reached. Higher powers may be computed only by discarding the last 3 or 4 figures in S.R.

Using the Model II the 4th power also may be computed without clearing S.R. and C.R. by developing the appropriate number in C.R. in a manner similar to that which has been described for the 3rd power, the cube being finally contained in C.R. For higher powers it is suggested that some convenient number (square, 3rd power, etc.) be set in S.R., in which case, when larger numbers are being dealt with, the necessity for curtailing the number is indicated in advance.

Commerce and Industry

Checking of Invoices and goods

a) Calculation of items and total amount in an invoice

(Only with CURTA Model II)

13 Articles at	1.48	=	19.24
25 Articles at	4.45	=	111.25
39 Articles at	7.25	=	282.75
31 Articles at	11.55	=	358.05
Total		=	771.29

Machine ready.

Set 13000000013 in S.R.

Develop 1.48 in C.R. with the handle. Set decimal points as follows: between positions 2 and 3 of C.R., between the 11th and 12th, and 2nd and 3rd positions of R.R. R.R. indicates in both its left and right hand ends the product **19.24,** i. e. the first item. Clear C.R. and finally **only the left hand end** of R.R. so that the number 19.24 remains in the right hand end of R.R.

S.R. 25000000025. Develop 4.45 in C.R. with the handle. The left hand end of R.R. now contains the second item: **111.25.** In the right hand end of R.R. the second item has been accumulated with the first: 130.49. Clear only C.R. and the left hand end of R.R. S.R. 39000000039. Develop 7.25 in C.R. with the handle. In the left hand end of R.R. there appears the third item **282.75** and in the right hand end the sum of the first three items 413.24. Clear only C.R. and the **left hand end** of R.R.

S.R. 31000000031. Develop 11.55 in C.R. with the handle. At the left hand of R.R. appears the fourth item **358.05** and at the right hand end the final total **771.29.** (Remark: For eventual percentage reduction or increase of the invoice we refer to the exercises on percentage calculations on page 24.)

b) Check on the final total of an invoice

If we wish to calculate the **final total** of an invoice, without determining the separate items, then it is unnecessary to set two numbers in S.R. and the calculation may be performed **on a CURTA Model I:**

38 Articles at 14.30 =	543.40
52 Articles at 23.75 =	1235.—
Gross total =	1778.40

deducted returns:

17 Articles at 12.80 =	217.60
Net total =	**1560.80**

Machine ready.

Set 14.30 at the right hand end of S.R. Set decimal points in S.R. between positions 2 and 3, and in R.R. between positions 2 and 3. Develop 38 in C.R. with the handle. **Clear only C.R.** Set 23.75 at the right hand end of S.R. and develop 52 in C.R. with the handle. R.R. indicates the **gross total 1778.40. Clear only C.R.** Reversing lever in the **lower position.** Set 12.80 at the right hand end of S.R. and turning the handle in its minus position develop the number 17 in positions 1 and 2 of the carriage. (Negative multiplication.) R.R. contains the **net total 1560.80.** (Return the reversing lever to its normal position.)

c) Checking the total and number of articles

(Only with CURTA Model II)

74 Articles at 32.25 =	2386.50
38 Articles at 19.40 =	737.20
Total 112 Articles =	3123.70

deducted returns:

13 Articles at 26.35 =	342.55
net 99 Articles; net tot. =	2781.15

Machine ready.

Set a 1 at the extreme left (i. e. with knob 11 of S.R., at the extreme right the number 32.25, thus S.R.: 100000032.25. Set decimal points in S.R. between knobs 2 and 3, and in R.R. between positions 2 and 3. Develop the number 74 with the handle. The multiplier 74 appears in C.R. as a check for R.R. At the left hand end of R.R. appears the number of articles 74 and at the left hand end the item 2386.50. **Clear only C.R.**

S.R.: 100000019.40. Develop 38 in C.R. with the handle. At the left hand of R.R. the number of articles (112) has been accumulated and at the right hand end the sum of the two items viz. 3123.70. **Clear only C.R.** Move the reversing lever to its **lower position.**

S.R. 100000026.35. By turning the handle in its minus position develop 13 in C.R. The results displayed in **R.R.** are at the **left hand end 99 articles net,** and at the **right hand end 2781.15 = net total.** (Return the reversing lever to its normal position.)

Percentage calculations

A. Percentage increase

The amount 378.65 is to be increased by 4.5 %.

$378.65 \times \frac{4.5}{100}$. Set the number 378.65 at the extreme right of S.R. and develop 4.5 in C.R. with the handle.

Set a decimal point between positions 5 and 6 of R.R. The increase 17.03925 (rounded to 17.04) can now be read in R.R.

Without clearing or resetting any number complete the number in C.R. to 104.5 (100 % + 4.5 %). The final amount 395.68925 now appears in R.R. rounded to 395.689.

B. Percentage decrease

12 % is to be deducted from the amount 735.—.

$735 \times \frac{12}{100}$. The multiplication is quite normal. Set a decimal point between positions 2 and 3 of R.R. The 12 % **decrease** can be read in R.R.: **88.20.**

Without clearing or resetting any number develop the number 88 (i. e. 100 %—12 % = 88 %) in C.R. by turning the handle in the required manner. R.R. now contains the **net amount 646.80.**

Calculations of this type in particular can be completed in one computation using a CURTA Model II. In the last example the numbers were relatively small and a CURTA I might have been used in this way. Set 88 at the extreme right and 12 at the extreme left of S.R. Set decimal points between positions 6 and 7, 8 and 9, and 2 and 3 of R.R.

Develop 735 in C.R. with the handle. The results appear in R.R., namely **on the left the decrease 88.20** and **on the right the net amount 646.80.**

With larger numbers there is always the danger that the results will overlap. We therefore recommend that when setting two numbers in S.R. a CURTA Model II which can accommodate 15 figures in R.R., be used.

C. Profit margin

Cost Price 1257.—; Selling Price 3840.—; Profit?

Profit as a percentage of the Selling Price?

Carriage in position 6. Set 1257 at the right hand end of S.R. One **minus turn.** Set 3840 at the right hand end of S.R. One plus turn. R.R. now indicates the **profit 2583.—.**

Carriage in position 5. With the reversing lever in its lower position perform the subtractive division 2583 ÷ 38.40 (= 1% of the Selling price). The result in C.R. is **67.266 %** and expresses the profit as a percentage of the Selling price. Return the reversing lever to its normal position. If, in the same example we wish to express the profit as a percentage of the cost price, the calculation is as follows:

Carriage in position 6. Set 3840 in S.R. One plus turn. Set 1257 in S.R. One minus turn.

Moving the reversing lever to its lower position the subtractive division 2583 ÷ 12.57 (= 1% of the cost price) is finally carried out. The result in C.R. is **205.489 %** (Return the reversing lever to its normal position).

D. Compound percentages

If we wish to calculate the same percentages of, or deductions from, a series of numbers (for example cost prices) it is advantageous to regard the percentage expression as ratios, and conduct the calculations as a series of multiplications.

For example suppose that on a number of cost prices we wish to calculate 57 % profit and finally to deduct 10 % rebate and 2 % discount. The appropriate ratios are as follows:

Profit = 57 %; Cost price + profit = **157 %**;

Rebate 18 % =

$$\frac{157 \times 18}{100 \times 100} = \mathbf{28.26\ \%.}$$

Net (deducting rebate) =

$$\frac{157 \times (100-18)}{100 \times 100} = \mathbf{128.74\ \%.}$$

Discount 2 % $\frac{128.74 \times 2}{100 \times 100} = \mathbf{2.575\ \%.}$

Net (deducting discount)

$$\frac{128.74 \times (100-2)}{100 \times 100} = \mathbf{126.165\ \%.}$$

We apply these to a cost price of 3755.—: 3755 is set in S.R. and the 6 ratios (in bold type) are developed in C.R. by turning the handle appropriately, the results being read one after the other as follows:

Cost price (in S.R.)	3755.—
Profit 57 %	2140.35
Cost price + profit	5895.35
Rebate 18 %	1061.16
Net (deducting rebate)	4834.19
Discount 2 %	96.69
Net (deducting discount) . . .	4737.50

E. Profit and loss

1. Financial year A: Profit 35676.—
   Financial year B: Profit 43217.—

By how many percent is the profit in year B greater than that in year A? We compute

$$\frac{43\,217 \times 100}{35\,676}$$

The result of the division 4321700 ÷ 35676 is **121.14 %**

35676 = 100 %; the **increase in profit** is thus the difference **21.44 %.** The net amount of the increase in profit **7541.—** may be determined in the same calculation as the difference 43217.— (R.R.) — 35676.— (S.R.), and occurs in the subtractive division after the first minus turn of the handle, and after the additive division it may be derived by carrying out one minus turn of the handle in the hundreds figure of the quotient in C.R.

2. Financial year A: Profit 17863.—
   Financial year B: Profit 14937.—

We wish to express the decrease in profits from year A to year B as a percentage. We must carry out the division 1493700 ÷ 17863.
The result in C.R. rounded to two decimal places is 83.62 %. The reduction is thus 100 %—83.62 % = **16.38 %.**

This result can be obtained **directly** in C.R. by positioning the reversing lever appropriately at the beginning of each division: in the lower position for additive division and in the upper position for subtractive division. These positionings of the reversing lever cause the quotient in C.R. to appear as a complement, and it thus may occur that the most significant positions of C.R. are filled with 9's which are not taken into account when reading off the result (e. g. 9/1638).

F. Capital and interest

1. The yearly interest on a capital investment of 67855.00 is 3912.00. What is the rate of interest?

We divide the interest 3912.00 by 1 % of the capital. The division 3912 ÷ 678.55 is carried out quite normally, and by inserting the appropriate decimal point in C.R. the result is 5.765 %.

2. The yearly interest on a capital investment is 7593.00 from an interest rate of 4.75 %. How much capital has been invested?

We divide the interest 7953.00 by 4.75, and multiply the result by 100. **Result 167432.00 = Capital investment.**

3. **The calculation of interest** by using the known formulae

$$\frac{\text{Capital} \times \text{rate of interest} \times \text{days}}{360 \times 100}$$

or

$$\frac{\text{Investment index} \times \text{rate of interest}}{360}$$

can be performed by using various Interest tables and one of the examples on pages 10—12.

Costing

The yearly expenditure in three departments are as follows

3545.00 in department A
6893.00 in department B
2360.00 in department C

in all: 12798.00.

What is the expenditure of each department expressed as a percentage of the total expenditure?

We proceed by dividing every expenditure by the total 12798.

Machine ready.

Set 12798 at the right hand end of S.R. The first division 3545 ÷ 12798 is carried out by the additive method starting with the carriage in position 5. After the subsequent division R.R. indicates 003545.04600 and C.R. **27.70 % = Proportion A.**

Do not clear. Starting with the carriage in position 5 the dividend 3545 is replaced by the new dividend 6893 by plus and minus turns of the handle. Thus with the carriage in

Position 5: 2 plus turns
Position 4: 6 plus turns
Position 3: 2 plus turns
Position 2: 4 minus turns

R.R. now contains 006893.00280,
C.R.: **53.86 %** = proportion B.

Do not clear. Starting with the carriage in position 5 the dividend 6893 is replaced by the new dividend 2360 by turning the handle in the appropriate manner. Thus with the carriage in

Position 5: 4 minus turns
Position 4: 5 plus turns
Position 3: 4 minus turns
Position 2: 2 minus turns

R.R. now contains 002359.95120,
C.R.: **18.44 % = proportion C.**

It is easy to check the calculation by adding the percentages together. The sum must, within the prescribed accuracy, be 100 %. By using the CURTA Model II it is possible to incorporate this check automatically as will be demonstrated in the following calculation (see the next example).

Remark: When a large number of items occur in the calculation, it is most economical to compute the reciprocal of the divisor and set this reciprocal in S.R. as a constant multiplier, and to develop the various items in C.R. (See Division by Multiplying by a Reciprocal on page 9).

Costing with simultaneous control

(Only with CURTA Model II)

The yearly expenditure in three departments are as follows:

3545.00 in department A
6893.00 in department B
2360.00 in department C

in all: 12798.00.

What is the expenditure of each department expressed as a percentage of the total expenditure?

Machine ready.

S.R.: 10000012798. Carriage in position 5. Divide into 3545 by the additive method. Thus with the carriage in

Position 5: 3 plus turns
Position 4: 2 minus turns
Position 3: 3 minus turns

The further passage to positions 2 and 1 is unnecessary in this case.
The percentage **27.70 % = proportional expenditure A** appears at the left hand end of R.R.

The same number appears in C.R. In order to check the calculation this number is **not cleared** so that the percentage may be accumulated in C.R. and be seen to total about 100 %.

Clear R.R. only. Carriage in position 5.

The division 6893 ÷ 12798 is again carried out by the additive method. At the left hand end of R.R. appears the percentage **53.86 % = proportional expenditure B.**

In C.R. the two percentages have been accumulated: 81.56 %.

Clear R.R. only. Commencing with the carriage in position 5 build up the next dividend 2360. The quantity 2359.95120 appears in R.R. as an approximation to 2360. At the right hand end of R.R. appears the percentage **18.44 % = proportional expenditure C. As a check** the sum of the three percentages appears in C.R.: **100 %.**

Calculations with nines transfer

From a product 96 × 2.35 (which might perhaps be gross wages) we wish to deduct a number of items (e. g. taxes, premium, contributions) namely 22.35, 8.74, 1.16.
Quantities required: the sum of the quan-

tities to be deducted, and the remaining net total.

Machine ready.

Obtain the product 96 × 2.35 by normal multiplication. The product stands at the extreme right of R.R.: 225.60. **Clear only C.R.** Set decimal points in R.R. between positions 2 and 3, 6 and 7 and 9 and 10. Set 9's with knobs 1 to 7 in S.R. (9999999). The first deduction is 22.35. Develop 22.35 in C.R. by means of plus turns, i. e. in carriage positions 1 to 4. C.R. 22.35. Simultaneously in R.R. the number 22.35 has been subtracted from 225.60 and has been built up in the left hand part of R.R. At the right hand end of R.R. appears the balance 203.25.

Clear only C.R. Develop the number 8.74 in C.R. in carriage positions 1 to 3. 8.74 is subtracted from the right hand part of R.R. and added to the left hand part, R.R. contains 310900194.51.

Clear only C.R. Develop 1.16 in the appropriate part of C.R. R.R. now contains **on the left the sum of the deductions: 32.25** and **on the right the net total 193.35.**

Exchange calculations

a) How many dollars may be obtained for 1200.00 units of a special currency E at an exchange rate of 1 $ = 4.22¼ E?
Machine ready.

The amount 1200 is divided by 4.2225.
The result in C.R. is **284.19 $.**

b) How much does $ 733.25 cost in E units at an exchange rate of 1 $ = 4.19½ E?
Machine ready.

We multiply the amount 733.25 by the rate of exchange 4.195. **The result** in R.R. is **3075.98.**

c) The dollar is quoted in Zurich at 4.30½ francs = 1 $. What is the parity in New York (i. e. how many dollars may be obtained at

Conversion table for pence to decimals of £ 1

Pence	d.	1/4 d.	1/2 d.	3/4 d.	Pence
0	.000 000 0	.001 041 7	.002 083 4	.003 125 0	0
1	.004 166 7	.005 208 4	.006 250 0	.007 291 7	1
2	.008 333 4	.009 375 0	.010 416 7	.011 458 4	2
3	.012 500 0	.013 541 7	.014 583 4	.015 625 0	3
4	.016 666 7	.017 708 3	.018 750 0	.019 791 7	4
5	.020 833 4	.021 875 0	.022 916 7	.023 958 4	5
6	.025 000 0	.026 041 7	.027 083 4	.028 125 0	6
7	.029 166 7	.030 208 4	.031 250 0	.032 291 7	7
8	.033 333 4	.034 375 0	.035 416 7	.036 458 4	8
9	.037 500 0	.038 541 7	.039 583 4	.040 625 0	9
10	.041 666 7	.042 708 4	.043 750 0	.044 791 7	10
11	.045 833 4	.046 875 0	.047 916 7	.048 958 4	11

the same exchange rate in New York for 100 francs).

Machine ready.

It suffices to calculate the price of 1 franc in dollars, i. e. 1 ÷ 4.305, and multiply the result by 100. The quotient of the division 1 ÷ 4.305 in C.R. is 0.232288. For 100 francs one may thus obtain $ 23.2288 in New York. The parity in New York is thus about **23.22⅞.**

Calculations with English currency

A) Multiplication and Division

In order to multiply or divide sterling amounts it is necessary to express shillings and pence as decimal fractions of a pound. **Shillings:** since 20 s = £ 1, 10 s = £ 0.5 and **1 s = £ 0.05. The number of shillings to be dealt with must thus be multiplied by 0.05,** e. g. 17 s = £ 17 × 0.05 = £ 0.85.

Pence: 1 s = 12 d; 20 s = 240 d; 1 d = £ 1/240 = **about £ 0.00417.** **The number of pence to be dealt with must thus be multiplied by 0.00417,** e. g. 3 d = £ 3 × 0.00417 = £ 0.0125. Except on special occasions, it is too irksome always to multiply the number of pence by 0.00417, and for this reason the ready reckoner opposite has been constructed, giving values of ¼ d to **11¾ d.**

Practical example: A gross of articles costs £ 17.13.7¼. What is the price of each article in sterling and in special currency E, the rate of exchange being £ 1 = 11.77 E?

Converting the shillings and pence into decimals of a pound:

	£	17
13 × 0.05	s	0.65
(Tables)	d	0.0302084
		17.6802084

Normal division of this quantity by 144 gives the price of one article in C.R. as 0.122779. £ 0.10 = 2 s, and from the table

£ 0.022779 = 5½ d. **The price of one article** is thus **2/5½.**

Clear R.R. only. Set the rate of exchange 11.77 in S.R. Place the reversing lever in its lower position. Starting with the carriage in position 1, reduce the figures in C.R. to zero by plus turns of the handle (see also pp. 11—12). The result in R.R., **1.44½**, is **the price of one article** expressed in currency E.

B. Addition and Subtraction

	£ 13.18. 9
+	£ 41.19.11
+	£ 7.17.10
=	£ 63.16. 6

Machine ready.

Set decimal points in S.R. and R.R. in front of the 3rd and 6th positions. Set the pence with knobs 2 and 1, the shillings with knobs 5 and 4, and the pounds with knobs 8 and 7 (13018009). After setting each amount carry out one plus turn. The addition gives in R.R. the amount **£ 61 54 s 30 d.**

Clear S.R. In order to divide the number of pence by 12, and at the same time add the quotient to the shillings, we set the decimal constant 988 in S.R. with knobs 3 to 1. After 2 plus turns 6 d remains. Clear S.R.

In order to divide the number of shillings by 20, and at the same time add the quotient to the pounds, we set the decimal constant 980 in S.R. with knobs 6 to 4. After 2 plus turns 16 s remains. **The sum in R.R. is £ 63.16.6.** (Return the reversing lever to its normal position.)

	£ 63.16. 6
—	£ 7.17.10
—	£ 41.19.11
=	£ 13.18. 9

Machine ready.

As a result of the previous calculation the first term is already installed in R.R. Set the 2nd amount in S.R. One minus turn. Similarly for the 3rd amount. R.R. now reads **£ 14/979/985.** Clear S.R. To convert the amount in R.R. to £, s, d, set 988 with knobs 3 to 1. **After 2 minus turns** there remains at the right of R.R. 9 d. Clear S.R. Set 980 with knobs 6 to 4. After 2 minus turns the shillings amount has also been converted. **The balance in R.R. is £ 13/18/9.**

For frequent sterling computations application of the CURTA Model II with its larger capacity is recommended.

Statistics

Simultaneous accumulation of a sum and a sum of squares

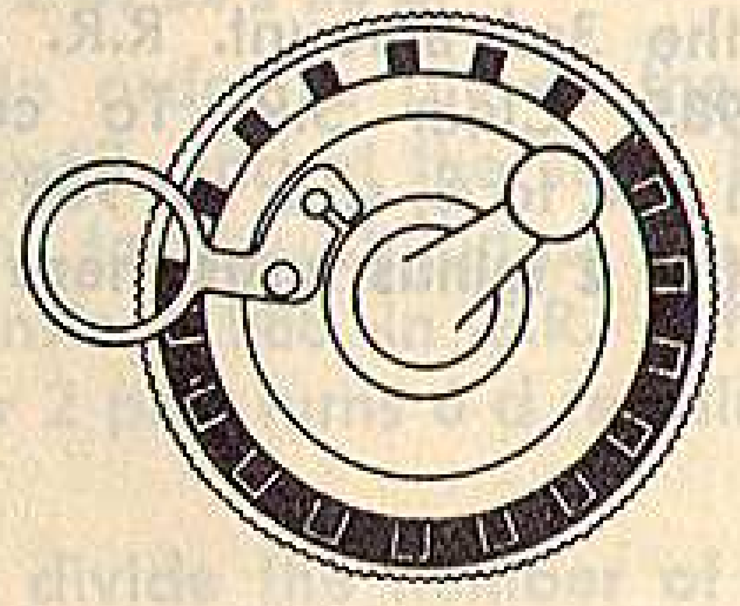
(Only with CURTA Model II)

$$a^2 + b^2 + c^2 + \ldots = S$$
$$a + b + c + \ldots = s$$

+ 6925	6925^2
+ 3289	3289^2
— 1721	1721^2
+ 2987	2987^2
s = ?	S = ?

Machine ready.

Move the clearing lever to the **left hand end of R.R. (see illustration).** Set decimal point between positions 11 and 10 of R.R. Set a 1 with knob 11 and the number 6925 with knobs 4 to 1 in S.R. Develop 6925 with the handle.



In C.R. there now appears **the number 6925,** on **the right of R.R. the square 47 955 625** and **on the left of R.R.** as a check **the multiplier 6925.** Clear **only the left hand end of R.R.;** i. e. move the clearing lever only over the number 6925 as far as the 0 **and return it to its original position.**
Set the next number, i. e. 3289, at the right hand end of S.R. (retaining the 1 in position 11). Develop 3289 with the handle so that the digit in S.R. generates the multiplier **at the left of R.R.** and acts as a check.

We have now accumulated the sum of the first two numbers in C.R., and the sum of the first two squares at **the right hand end** of R.R. As a check the multiplier 3289 is at **the left hand end** of R.R. Again we **clear only multiplier 3289 at the left hand end** of R.R. Set the next number at the right hand end of R.R. Since the sum is being accumulated in C.R. we move the **reversing lever to its lower position.** Develop the number 1721 normally with the handle. Again clear only the left hand end of R.R., (i. e. the multiplier 1721). Move the **reversing lever to its normal position.** Set the next number at the right of S.R. and multiply it by itself.

The final result at the right hand end of R.R. is **70 657 156 = S,** and in C.R. we have **11 480 = s.**

At the left of R.R. the last number 2987 stands as a check. You may again partially clear so that the sum S remains by itself for further calculations.

Computation of arithmetic mean and standard deviation

Given N observations $x_1, x_2, \ldots x_n$. The arithmetic mean is given by

$$\overline{x} = \frac{\Sigma (x_i - x_0)}{N}$$

and the standard deviation by

$$\overline{\Delta x} = \pm \sqrt{\frac{\Sigma (x_i - \overline{x})^2}{N (N-1)}}$$

In order to facilitate the calculation we reduce each observation by a known constant x_0 and in this manner reduce the number of figures in the numbers used in the calculation. We have

$$\overline{x} = x_0 + \frac{\Sigma (x_i - x_0)}{N} \quad \text{and}$$

$$\Sigma (x_i - \overline{x})^2 = \Sigma (x_i - x_0)^2 - N (\overline{x} - x_0)^2$$

The calculation can be carried out by the CURTA in the following manner, where the capacity of the Model II is always sufficient. If the number of figures in, and the dispersion of, the observations are not too large, the CURTA Model I may also be used, as the following example indicates:

Observations: $x_1 = 215.3$
$x_2 = 216.4$
$x_3 = 214.7$
$x_4 = 217.1$
$x_5 = 213.8$
$x_6 = 217.3$
$x_7 = 216.6$

We proceed with $x_0 = 210$; thus $x_1 - x_0 = 5.3$; $x_2 - x_0 = 6.4$, and so on.

Machine ready.
S.R.: 005.30001; **Carriage in position 3.**
Multiply by 5.3 and **clear only C.R.**

To mark the decimal points always use the white movable markers.
S.R.: 006.40001; **Carriage in position 3.**
Multiply by 6.4. Clear only C.R. After these two steps we have in R.R.:

069.05/0/11.700

i. e.

$(x_1 - x_0)^2 + (x_2 - x_0)^2 / (x_1 - x_0) + (x_2 - x_0)$

Proceed in this manner, and after 7 such steps we have in R.R.

252,84/0/41,200

or $\Sigma (x_i - x_0)^2 / \Sigma (x_i - x_0)$

Clear only C.R.
Set in S.R. $\Sigma (x_i - x_0)$ and N, thus: 041.20007.
Carry out the division 41.2 ÷ 7 in the right hand part of R.R. by the subtractive method. Thus move the reversing lever to its lower position and commence the division with the carriage in position 4. When the division is finished C.R. indicates 005.885, that is

$$\frac{\Sigma (x_i - x_0)}{N}$$

The mean value of the observations is thus

$x = 210 + 5.885 =$ **215.885**

R.R. contains 010.37800005. Up to the last 5 (which is the remainder of the division) this is

$$\Sigma (x_i - x_0)^2 - N(\bar{x} - x_0)^2 = \Sigma (x_i - \bar{x})^2$$

Clear only C.R., Carriage in position 6.
S.R.: Set N (N — 1), i. e. 042.00000, in S.R. Carry out the division

$$\frac{\Sigma (x_i - \bar{x})^2}{N (N - 1)}$$

by the subtractive method
Result in C.R.: 0.247.

By means of a slide rule or a CURTA the square root of this number is found to be 0.497. The observation measurement reads finally

$$\mathbf{x = 215.885 \pm 0.497.}$$

(Return the reversing lever to its normal position.)

Technical and Survey calculations

Division into a negative number (complementary number)

$$\frac{(a \times b) - (c \times d)}{e}$$

where $(a \times b) < (c \times d)$. e. g.

$$\frac{(3.15 \times 17.5) - (9.6 \times 23.3)}{137.4}$$

$(a \times b) - (c \times d)$:

Machine ready.

S.R. Set 3.15 at the extreme right. Set a decimal point in front of knob 2 of S.R.

Carriage in position 4: Normal multiplication by 17.5 in position 4, 5 and 6 of the carriage.

R.R.: . . . 55.125 . . . Set decimal points in front of position 4 in C.R. and position 6 of R.R.

Clear only C.R.

S.R.: Set 9.6 with knobs 3 and 2. Set knob 1 to zero. Place the reversing lever in its lower position.

Develop 23.3 in C.R. by turning the handle in **its minus position** in positions 4, 5 and 6 of the carriage. (Negative multiplication).

R.R.: . . . 99831.445 . . . = Complement of the dividend. **Do not clear R.R.**

Division by e

This division is carried out by building up the dividend to zero by means of the divisor. Return the **reversing lever to its normal position.** Clear only C.R. Carriage in position 6.

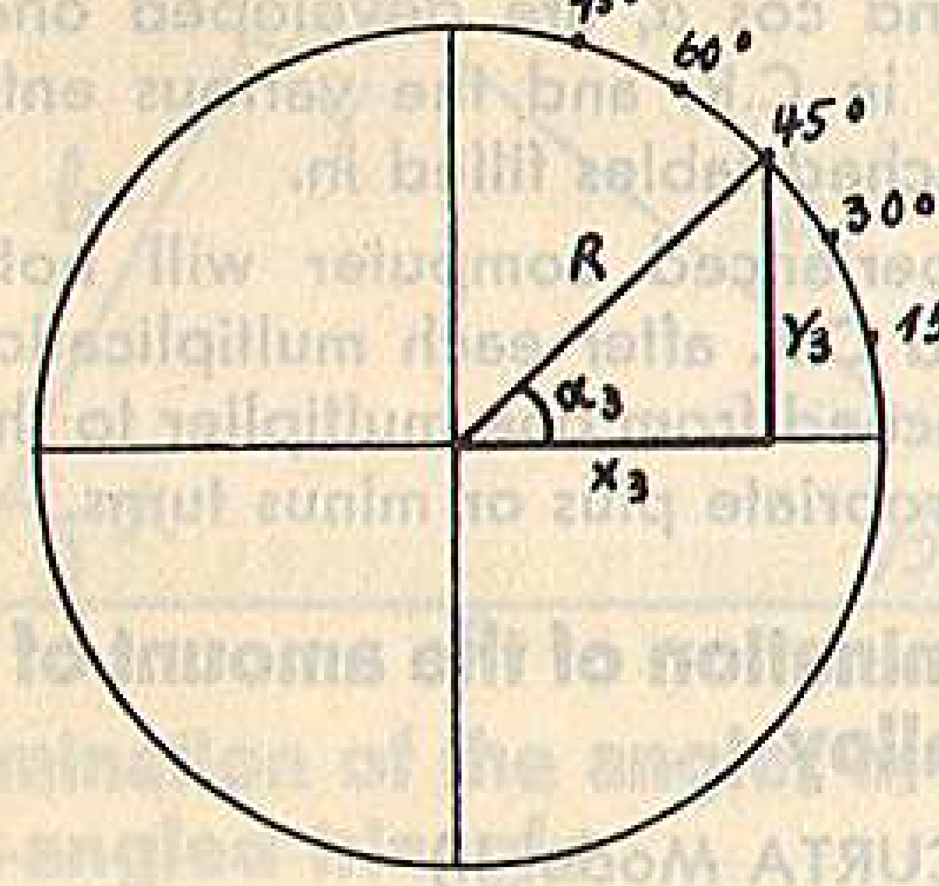
Set 137.4 in S.R. with knobs 4 to 1. Set decimal points in front of knob 1 in S.R. and in front of position 5 of C.R. By turning the handle force the contents of R.R. as near **zero** as possible, i. e. to 9999 etc.). Thus observe R.R. and in

Carriage position 6: 1 plus turn
Carriage position 5: 2 plus turns
Carriage position 4: 2 plus turns
Carriage position 3: 7 plus turns
Carriage position 2: 3 minus turns
Carriage position 1: 5 plus turns

There is now in R.R. the nearest possible approximation to zero (0.000450). The answer **—1.22675** stands in C.R.

Calculation of co-ordinates

Given: R = 21.7; $\alpha = 15°, \ldots, 75°$.
To find: $x_1, x_2, \ldots x_5;\ y_1, y_2, \ldots, y_5$.
Set R = 21.7 in S.R. with knobs 3 to 1 as



a constant multiplicand. By means of the handle the multipliers, i. e. the values of

Point	α	cos α	x = R. cos α	sin. α	y = R. sin. α
1	15°	0.96593	20.960681	0.25882	5.616394
2	30°	0.86603	18.792851	0.50000	10.850000
3	45°	0.70711	15.344287	0.70711	15.344287
4	60°	0.50000	10.850000	0.86603	18.792851
5	75°	0.25882	5.616394	0.96593	20.960681

sin α and cos α, are developed one after another in C.R. and the various entries in the attached tables filled in.

An experienced computer will not clear R.R. and C.R. after each multiplication, but will proceed from one multiplier to the next by appropriate plus or minus turns.

Determination of the amount of silver in an alloy

(Only CURTA Model II)

The ore is dissolved in HNO_3 and the solution mixed with HCl. After filtering and annealing we have, let us suppose, the following numerical data.

Q = Scale	=	10.0134
To = Weight of empty container	=	13.5627
Tn = Weight of full container	=	13.7434
Ag (Atomic Weight)	=	107.880
AgCl (Molecular Weight)	=	143.337

The calculation is based on the formula

$$x = \frac{100\ Ag \times (T_n - T_o)\ \%\ \text{Silver,}}{AgCl \times q} \quad \text{or}$$

$$x = \frac{100 \times 107.880 \times (13.7434 - 13.5627)}{143.337 \times 10.0134}$$

Machine ready.

(To mark the decimal points always use the white movable decimal markers.)

Difference T_n—T_o :

S.R. ... 13.7434. Carriage in position 8, 1 plus turn.

S.R. ... 13.5627. Carriage in position 8, 1 minus turn.

R.R. contains: 0000.18070000000.

Division by 10.0134

S.R. ... 10.0134. Set the reversing lever to its lower position. Carriage in position 6. Carry out subtractive division from figure to figure.

C.R. contains: 0.0180458 and

R.R.: 0000.000001868 (remainder).

Multiplication by 107.880

Clear R.R. only. Leave the reversing lever in its lower position. Carriage in position 1. S.R. . . . 107.880. By turning the handle in its plus position reduce the contents of C.R. to zero (see pp. 11 and 12).

R.R.: 00001.9467809040 and C.R.: 00000000.

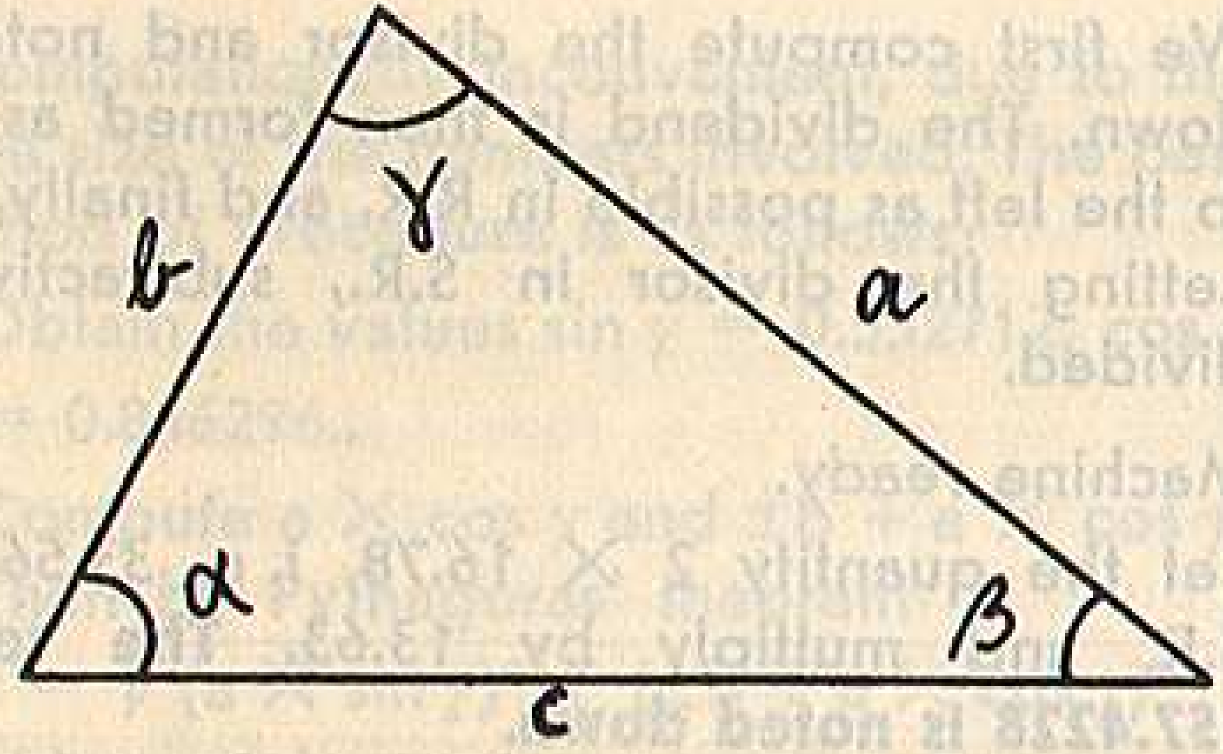
Division by 143.337

S.R. . . . 143.337. Carriage remains in position 6. Carry out subtractive division from figure to figure.

C.R. contains 0.0135818.
R.R.: 00000.0000064374 (remainder).

The multiplication by 100 is carried out by shifting the decimal point two places. The result is thus: **1.358(18) % Silver.**

(Return the reversing lever to its normal position.)



Determination of the angles in an acute-angled triangle

given three sides (only CURTA Model II)

Given: $a = 16.78$
$b = 13.63$
$c = 20.33$

To find: γ.

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2\,a\,b}$$

$$\cos \gamma = \frac{16.78^2 + 13.63^2 - 20.33^2}{2 \times 16.78 \times 13.63}$$

We first compute the divisor and note it down. The dividend is then formed as far to the left as possible in R.R. and finally, by setting the divisor in S.R., subtractively divided.

Machine ready.

Set the quantity 2 × 16.78, i. e. 33.56, in S.R. and multiply by 13.63. **The result 457.4228 is noted down.**

Machine ready.

Set the number 16.78 in S.R. by means of knobs 8 to 5: 00016780000. Develop 16.78 in C.R. as follows:

Carriage in position 5: 2 minus turns
Carriage in position 6: 2 minus turns
Carriage in position 7: 2 minus turns
Carriage in position 8: 2 plus turns.

Clear C.R. only. Set a decimal point between positions 12 and 13 of R.R. Set the number 13.63 in S.R. with knobs 8 to 5: 00013630000, and develop the number 13.63 in C.R. in positions 5 to 8. R.R. now contains the sum of the first two squares: 467.3453.

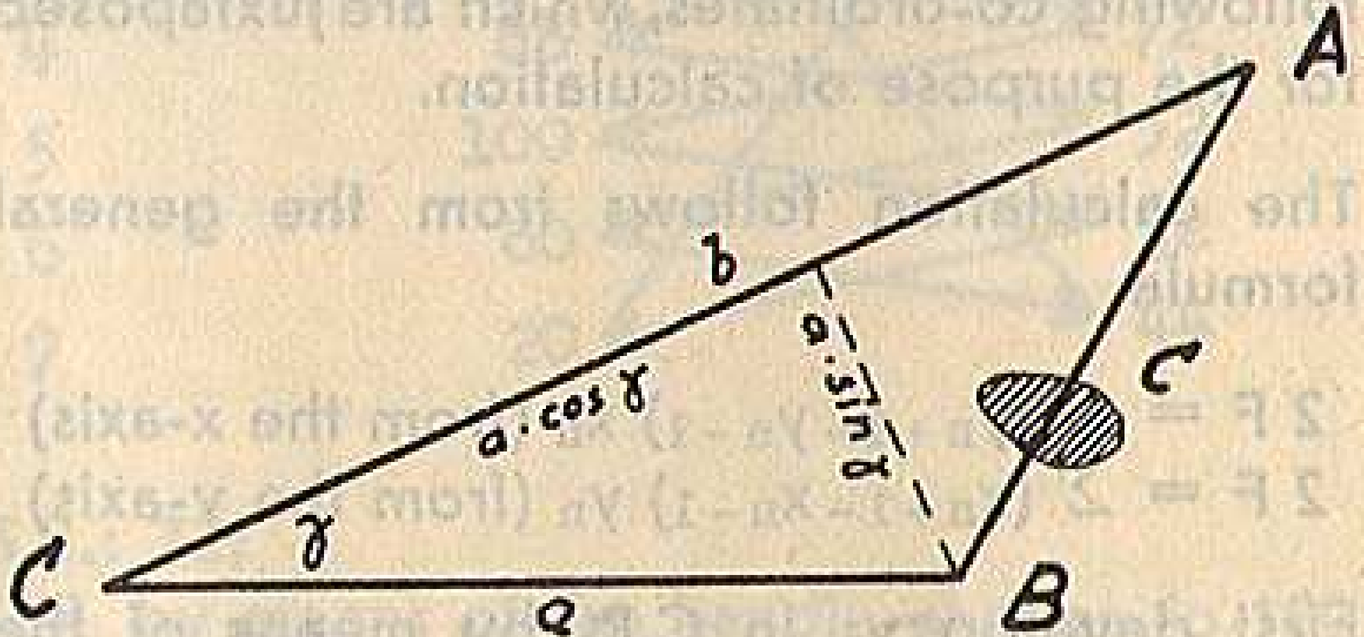
Clear C.R. only. Set the number 20.33 in S.R. with knobs 8 to 5: 00020330000. Place the reversing lever **in its lower position.** Develop 20.33 in C.R. by turning the handle **in its minus position** in positions 5 to 8 of the carriage (negative multiplication). In R.R. the third square has been subtracted from the sum of the first two and the result is the **dividend 54.0364.**

Clear C.R. only. Place the carriage in position 8. Set the divisor 457.4228 (previously noted down) in S.R. with knobs 7 to 1: 00004574228, and set a decimal point between knobs 5 and 4. Now follows the division by the subtractive method (see page 7).

The final result in C.R. after applying the decimal point rule is **0.11813228 = cos γ.** (Return the reversing lever to its normal position.)

Determination of a side of an obtuse-angled triangle

(given the two other sides and their included angle)



The distance A—B is unknown, or it is perhaps inconvenient to measure it.

Given: a = 21.47 in.
b = 32.14 in.
γ = 32° 11′ 20″
To find c = ?

Direct use of the classical formula

$$(c^2 = a^2 + b^2 - 2a \times b \times \cos\gamma)$$

is computationally inconvenient, due to the large size of the numbers involved. The best method of procedure is as follows:

1. Obtain the values $\sin\gamma = 0.532712$, $\cos\gamma = 0.846296..$
2. Compute $a \times \cos\gamma$ and $\pm b \mp a \times \cos\gamma$.
3. Then by Pythagoras' theorem
$$c = \sqrt{(a \times \sin\gamma)^2 + (\pm b \mp a \times \cos\gamma)^2}$$

Machine ready.

To mark the decimal points always use the white movable markers.

S.R.: 21.47. Develop 0.532712 in C.R. with the handle.

R.R.: **a × sin γ = 11.437.** Note this number down and **do not clear.**

C.R.: By turning the handle change the contents to 0.846296.

R.R. **a × cos γ = 18.16997 . . ., not cleared.** Carriage position 6.

S.R.: Set b = . . . 32.14 to correspond with the number in R.R.

Handle: 1 minus turn. If $b > a \times \cos \gamma$, a complement appears in R.R., which is the case with the present example.

S.R.: Set the modulus of the number in R.R. (If $b < a \times \cos \gamma$, then set the number in R.R.) In the present example the number (rounded off) is . . . 13.970. As a check carry out one plus turn when R.R. reads (1)0000... or 9999 . . . Clear R.R. and C.R. Finally multiply the number in S.R. by itself, thus 13.970 × 13.970.

R.R.: 195.160900 . . . $= (\pm b \mp a \times b \cos \gamma)^2$. Clear C.R. only.

S.R. 11.437 $= a \times \sin \gamma$. Multiply by 11.437.

R.R.: 325.965869 $= (a \times \sin \gamma)^2 + (\pm b \mp a \times \cos \gamma)^2$.

Now compute the square root by Hermann's (see page 13) or any other appropriate method.

The result is **18.054 in. = c.**

Calculation of area from co-ordinates

(By Elling's Method)

The area to be computed is defined by the following co-ordinates, which are juxtaposed for the purpose of calculation.

The calculation follows from the general formula

$$2F = \Sigma (y_{n+1} - y_{n-1})\, x_n \text{ (from the x-axis)}$$
$$2F = \Sigma (x_{n+1} - x_{n-1})\, y_n \text{ (from the y-axis)}$$

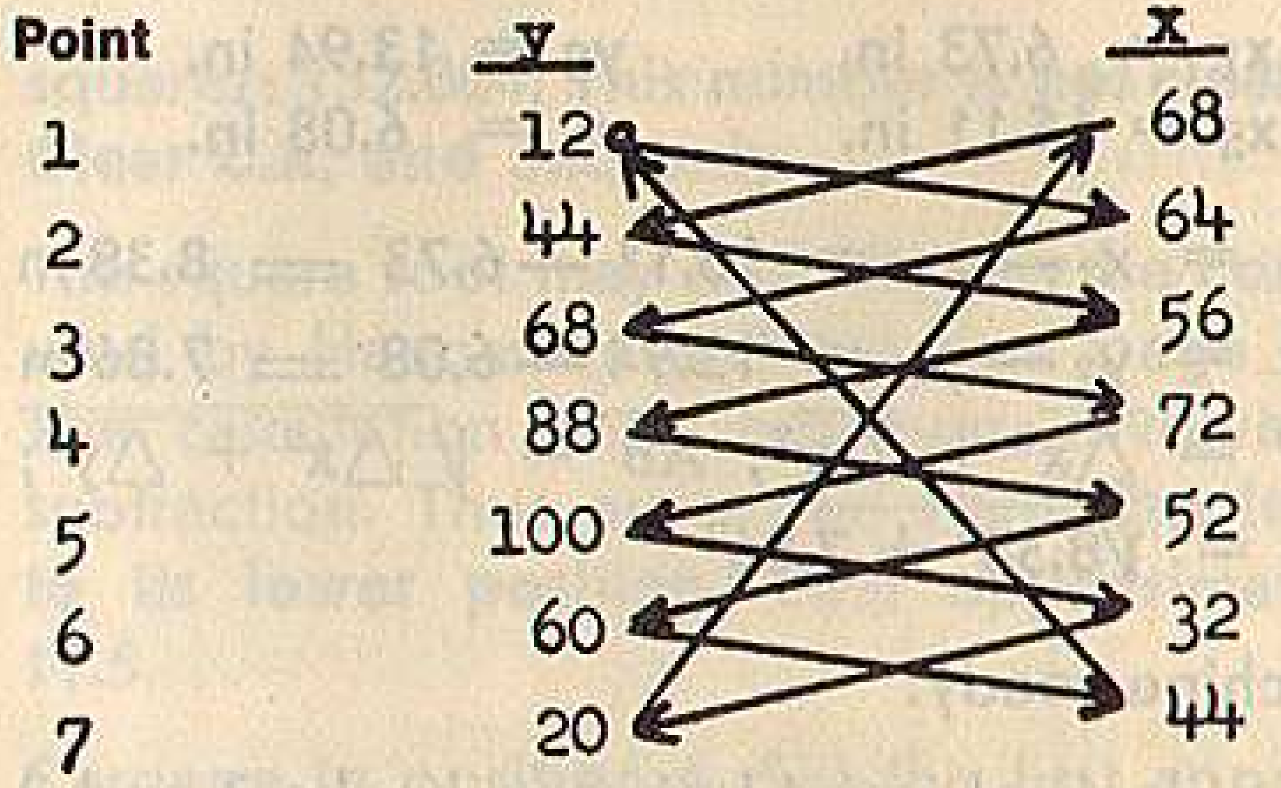
First develop y_1 in C.R. by means of the handle. Now set x_2 in S.R., x_2 must be multiplied by $(y_1 - y_3)$, thus

C.R.: develop 12.

S.R.: Set 64 at the extreme right.

C.R.: By means of the handle change to 68 (difference between y_1 and y_3).

The arrows in the following scheme indicate the further course of the calculation.



S.R.: 72;
C.R. by means of the handle change to 100

S.R.: 32;
C.R. by means of the handle change to 20

S.R.: 68;
C.R. by means of the handle change to 44

S.R.: 56;
C.R. by means of the handle change to 88

S.R.: 52
C.R. by means of the handle change to 60

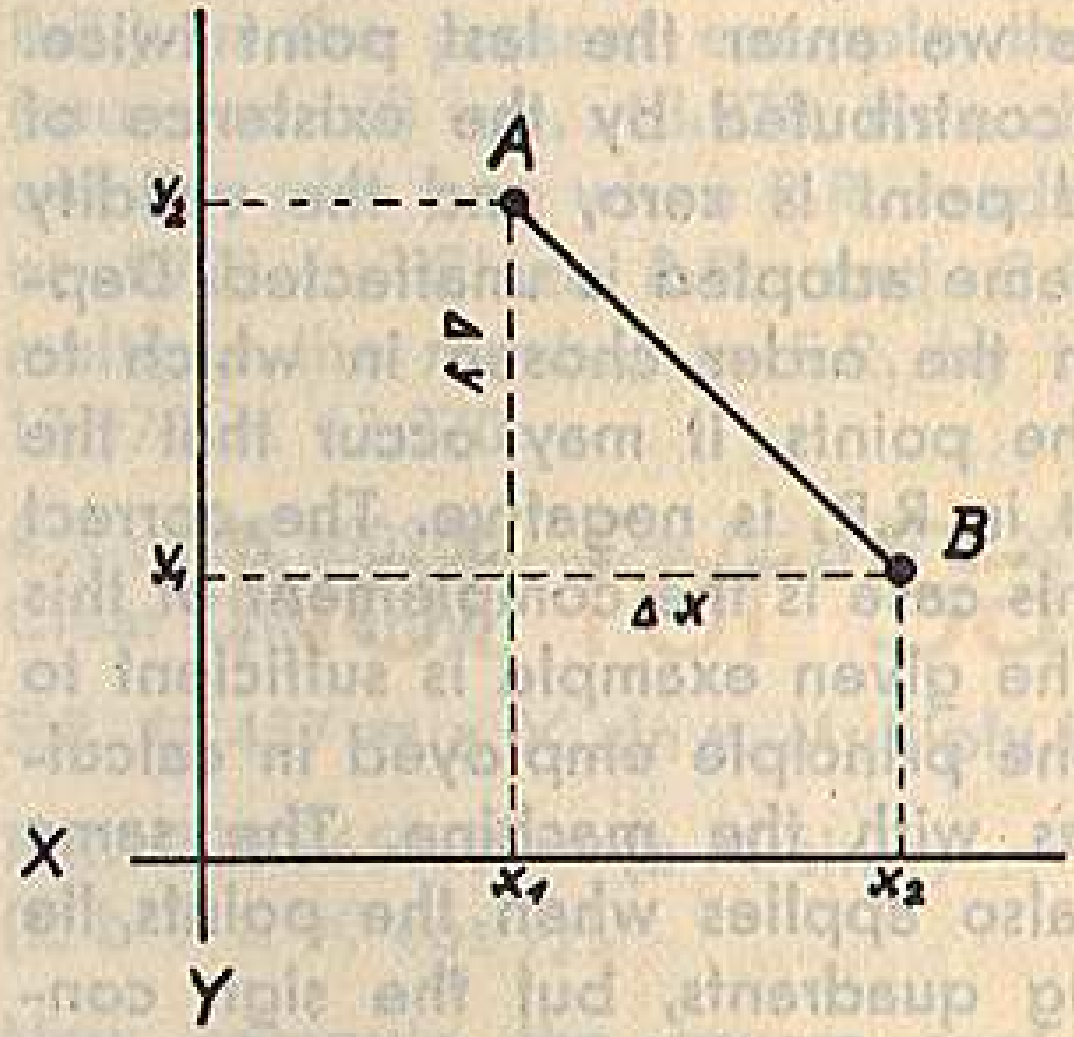
S.R.: 44;
C.R. by means of the handle change to 12

R.R. contains 3856 = 2F. **F = 1928.**

If the number of points given is even, then in practice we enter the last point twice. The area contributed by the existence of the added point is zero, and the validity of the scheme adopted is unaffected. Depending on the order chosen in which to traverse the points, it may occur that the final result in R.R. is negative. The correct result in this case is the complement of this number. The given example is sufficient to establish the principle employed in calculating areas with the machine. The same principle also applies when the points lie in differing quadrants, but the sign convention must very strictly be obeyed.

Calculation of the distance between two points

given their co-ordinates
(Using Pythagoras' theorem)



The co-ordinates of the points A and B are known. We wish to find the distance AB.

A: $x_1 = 6.73$ in. $\quad y_2 = 13.94$ in.
B: $x_2 = 15.11$ in. $\quad y_1 = 6.08$ in.

$$\overline{\Delta x} = x_2 - x_1 = 15.11 - 6.73 = 8.38 \text{ m}$$
$$\overline{\Delta y} = y_2 - y_1 = 13.94 - 6.08 = 7.86 \text{ m}$$
$$\overline{AB}^2 = \overline{\Delta x}^2 + \overline{\Delta y}^2; \quad \overline{AB} = \sqrt{\overline{\Delta x}^2 + \overline{\Delta y}^2};$$
$$\overline{AB} = \sqrt{8.38^2 + 7.86^2}$$

Machine ready.

Set a decimal point between knobs 3 and 2 of S.R. Set 8.38 at the right of S.R. With the carriage in positions 4 to 6 develop the number 8.38 in C.R. by means of the handle. Set a decimal point between position 8 and 7 of R.R. The product in R.R., 70.2244, will be accumulated with the next product (7.86 × 7.86), and is therefore not cleared. **Clear C.R. only.**

Set 7.86 at the right of S.R. With the carriage in positions 4 to 6 develop the number 7.86 in C.R. by means of the handle. R.R. now contains the sum of the two squares: 132.004. This number is **not cleared. Clear C.R. and S.R. only.**

The square root may now directly be computed by Hermann's method (see page 13). The calculation here, however, involves a subtraction. Thus move the **reversing lever to its lower position.** First approximation 11.5.

Carriage in position 6. Set the first approximation 11.5 in S.R. with knobs 5 to 3. Develop the number 11.5 in C.R. by turning the handle in its minus position with the carriage in positions 6, 5 and 4.

S.R.: Change knobs 5 to 3 to 230 and in positions 3, 2 and 1 of the carriage use the handle to bring the number in R.R. as close to zero as possible. C.R. then indicates the square root 11.4893 (R.R. contains 0.0001). The length AB = 11.49 in.

(Return the reversing lever to its lower position.)

Calculation of Distance and Azimuth between two points with known Co-ordinates: P_A (Y_A X_A), P_B (Y_B X_B)

(Only CURTA Model II)

Formulae: $\tan \varphi = \frac{(Y_B - Y_A)}{(X_B - X_A)}$; $D = \frac{(X_B - X_A)}{\cos. \varphi}$

Co-ordinates:

	Y	X	
P_B	+ 94892.791	—11779.323	(3rd Quadrant)
P_A	+ 95141.42	—11517.15	

1: Y_B—Y_A

Machine ready!

(To mark the decimal points always use the white movable markers.)

Carriage: position 7.

S.R.: 00094892.791 1 plus turn
S.R.: 00095141.420 1 minus turn
R.R.: 999751.371 . . . = **complement of** Y_B—Y_A.

Do not clear R.R.

2: $X_B—X_A$

Carriage: position 1.

S.R.: 00011779.323 1 minus turn
S.R.: 00011517.150 1 plus turn
R.R.: 999751.371* / 737827 /

737827 = complement of $X_B—X_A$.

S.R.... 262.173 = **absolute value of $X_B—X_A$.**

1 plus turn.

R.R.: 999751.371... = **complement of $Y_B—Y_A$.**

Clear C.R. Note down the **absolute value of** 248.629.

3: $\dfrac{Y_B—Y_A}{X_B—X_A}$

(Division by using the divisor to bring the contents of R.R. up to zero (see page 40). Reversing lever **in its normal position.**

Carriage in position 7: 1 plus turn
Carriage in position 6: 1 minus turn
Carriage in position 5: 5 plus turns
Carriage in position 4: 2 minus turns
Carriage in position 3: 4 plus turns
Carriage in position 2: 7 minus turns
Carriage in position 1: 9 plus turns

C.R.: 00.948339 = tan φ; φ (from tables = **248g 31c 24cc** cos φ = 0.725601.

Do not clear S.R.

4: $\dfrac{X_B—X_A}{\cos\varphi}$

Clear C.R. and R.R. only.
Carriage in position 7. 1 plus turn.
R.R.: 000262.173 ... = Dividend $X_B—X_A$.
S.R.: ...0.725601. Carriage in position 6.
Clear C.R.
Move the reversing lever to its lower position.
Carry out a substractive division.
C.R.: **000361.318 = D.**

* Use two contiguous markers here.

5: $\frac{Y_B - Y_A}{\sin\varphi}$ (as a check)

$\sin\varphi = 0.688116$. Machine ready.
Carry out the division 248.629 ÷ 0.688116.
C.R.: 361.31844. **D = 361.318.**

Linear interpolation

One can obtain two values Y_1 and Y_2 of a function by looking in tables. Suppose that we are interested in an intermediate value Y_n. This may be determined by using the interpolation formula,

$Y_n = Y_1 + (Y_2 - Y_1) \times n$. For using the CURTA this is conveniently rearranged as:

$$Y_n = Y_2 \times n + Y_1 (1 - n).$$

Example: to find sin 17° 14′.

From tables: sin 17° = 0.29237 and sin 18° = 0.30902. Since the argument is in steps of 1° = 60′, we must interpolate using as argument 14/60 = 0.233. (An experienced computer will carry out the division by 60 in his head.)

Set Y_2 = sin 18° = 0.30902 at the right hand end of S.R., and in C.R. a decimal point between positions 3 and 4. Develop 0.233 in C.R. with the handle. **Clear neither R.R. or C.R.**

Set Y_1 = sin 17° = 0.29237 in S.R., and complete the number in C.R. to 1.000 (multiplication by 1—0.233). We find the required value in R.R.: 0.29624(945). Thus, rounding to 5 figures, we have

sin 17° 14′ = 0.29625.

R.R.: 999751.371 [illegible]

[illegible]

(Division by using the divisor to bring the contents of R.R. up to zero (see page 49). Reversing lever in its normal position.

[3] Use two contiguous markers here.

Carriage in position 7: 1 plus turn
Carriage in position 6: 1 minus turn
Carriage in position [illegible]
Carriage in position [illegible]
Carriage in position [illegible]
Carriage in position 2: 7 minus turns
Carriage in position 1: 9 plus turns

C.R.: [illegible] = tan φ (from tables)

[illegible]

R.R.: [illegible] = Dividend $X_B - X_A$

S.R.: [illegible] Carriage in position 6.

Clear C.R.

Move the reversing lever to its lower position.

Carry out a subtractive division.

C.R.: [illegible] = φ.